Dublin, October 2010

A Constitutive Framework for the Numerical Analysis of Organic Soils and Directionally Dependent Materials

FracMan Technology Group

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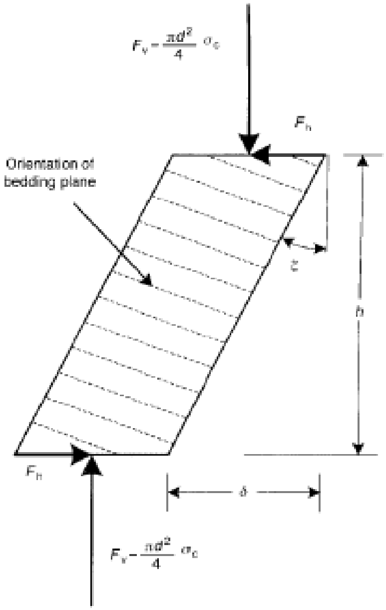
Golder Associates

Presentation Outline

- Some Physical Observations of Organics;
  - Fibrous nature;
  - Design Requirements; and
  - Laboratory Testing.
- Numerical Modelling
  - Its basic origins;
  - Material types
- A Generalised Numerical Constitutive Framework;
  - Applicability;
  - Basic mechanical strength envelope;
  - Creep and Rate Dependency;
  - Inclusion of Orthotropy; and
  - Porous Flow & Thermal dependency.

Some Features of Organic Soils

- Mixture of fragmented organic material formed in wetlands ;
- The soil has essentially an open structure with interstices filled with a secondary structural arrangement of nonwoody, fine fibrous material. If >20% fibre content classified as fibrous;
- Fibrous peat differs from amorphous peat in that it has a low degree of decomposition and easily recognizable structure;
- The compressibility of fibrous peat is very high and so it’s rate of consolidation;
- Formation of peat deposits leads to a pronounced structural anisotropy in which the fibres tend to have horizontal orientation; and
- Under a consolidation, deformation is directionally dependent and water tends to flow faster from the soil in the horizontal direction than in the vertical direction.



Design Requirements

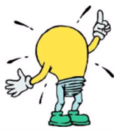
- When designing a foundation for an embankment its influence on its surroundings is often an important issue;
- In considering this influence a proper indication of horizontal deformations and horizontal stresses in the subsoil is needed;
- Many sediment deposits are deposited in horizontal strata, so it is to be expected that their mechanical properties in both horizontal directions might differ from their properties in vertical direction;
- It is widely recognized that soft soil might show anisotropic behaviour;
- Soft soil is often described as an anisotropic heterogeneous material. Each soil property like permeability, stiffness or strength might show anisotropic behaviour.

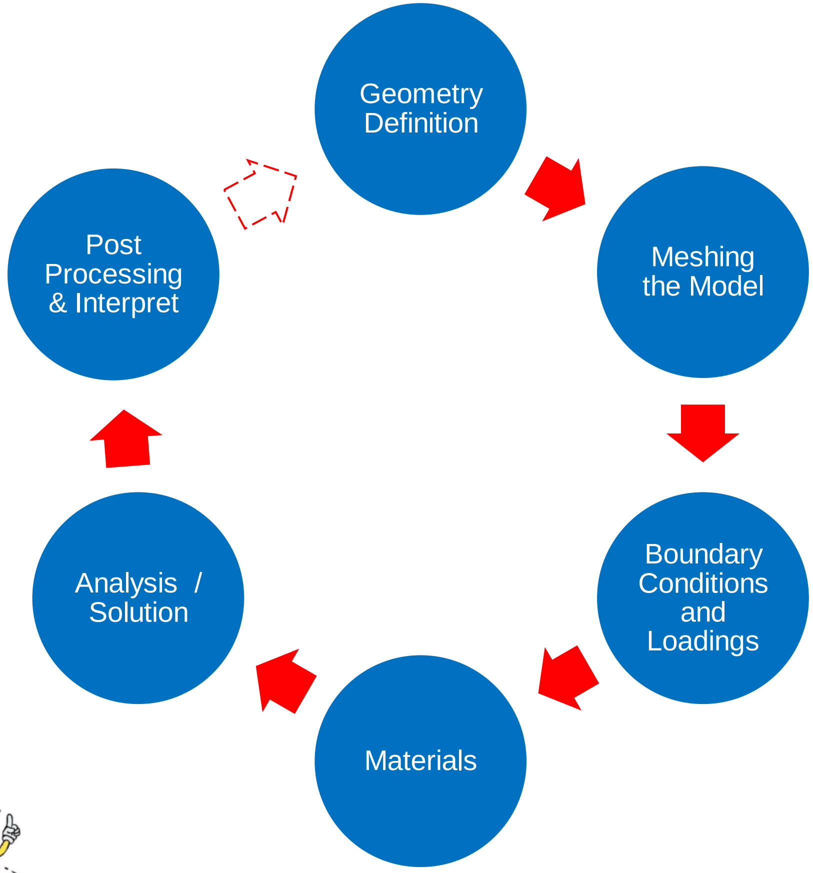
Laboratory Testing

- Laboratory testing methods to test for the shear strength of peat are generally the same as for traditional soils.
- Triaxial compression (TXC), direct shear (DS), direct simple shear (DSS) and ring shear (RS) have been used to measure undrained and effective strength properties.
- Laboratory testing of peat strength properties is complicated by several factors, as follows.
  - It is difficult to obtain and prepare samples because of the high water content and fibres.
  - Corrections related to apparatus compliance and membrane stiffness can be a large percentage of the measured strength.
  - Interpretation of actual failure is difficult because of the large strains involved and excessive deformation.
  - Structural anisotropy as a result of the presence of fibres within peat can cause artificial reinforcement of the sample

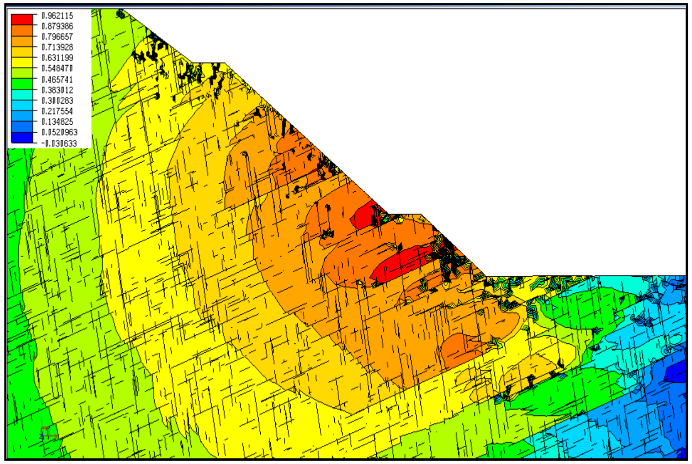
Numerical Modelling

- **What is Numerical Modelling?**
  - *Wikipedia (2009), "A computer simulation, a computer model or a computational model . . . . that attempts to simulate a particular system. Computer simulations have become useful in the process of engineering, . . . to gain insight into the operation of those systems, or to observe or understand their behaviour or limits."*
- **Types of Numerical Modelling?**
  - Semi analytical approximations;
  - Stress modelling – continuous/discontinuous
- **What they all have in common is .**
  - They all look to solve Complex Equations that bound a specific problem

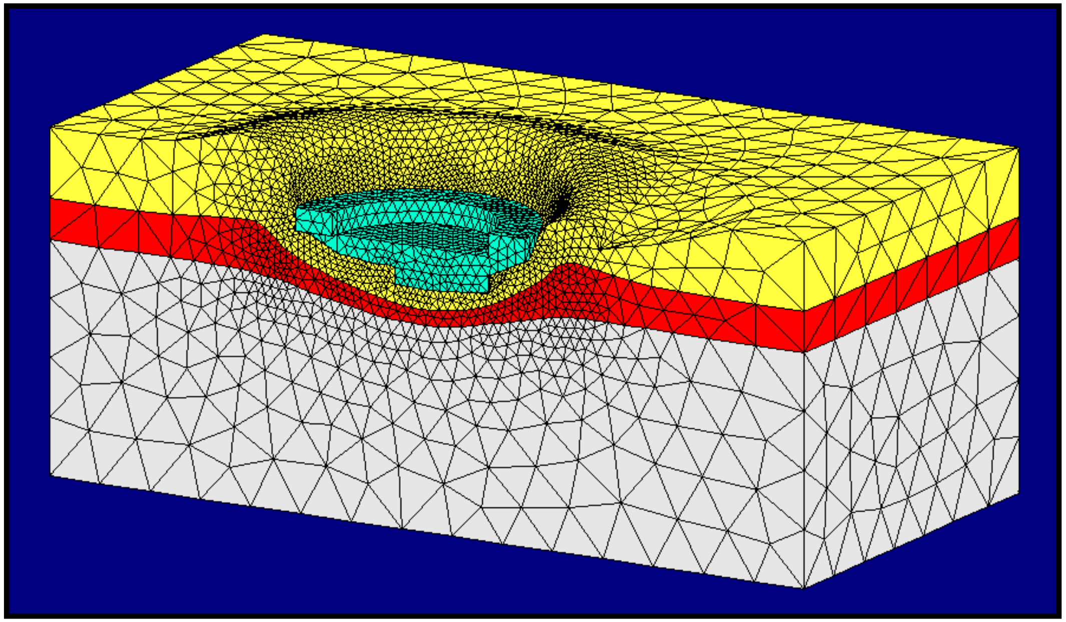




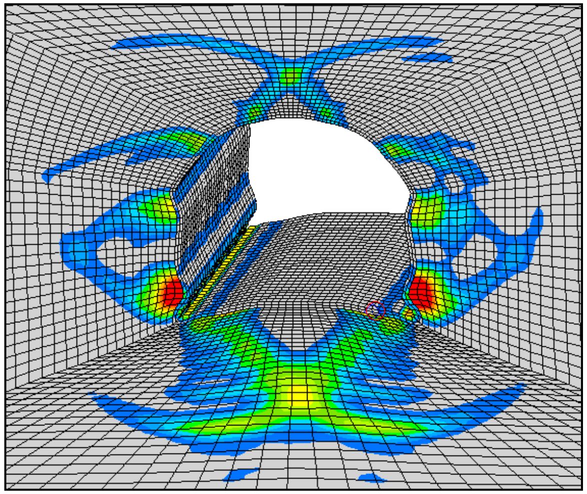
Geomechanical Modelling



Slope Stability



Soft soil foundations



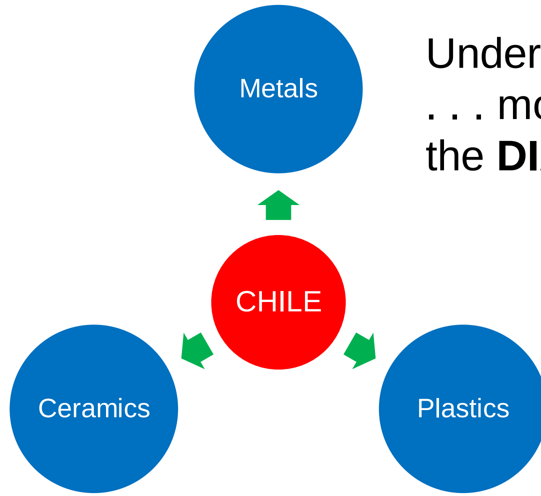
Tunnel Integrity

Material Characterisation

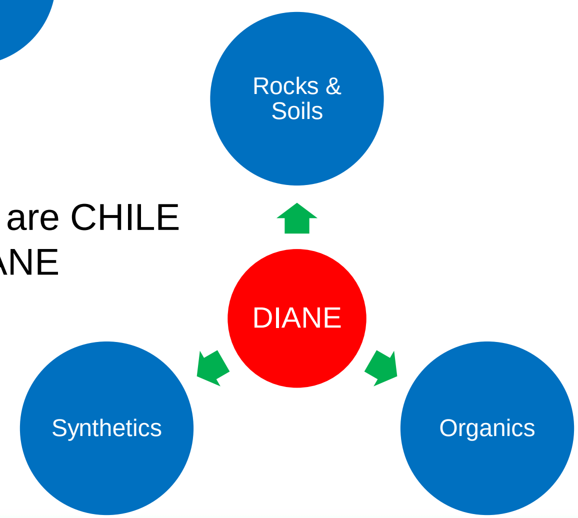
- **Categories of Material**
  - **C**ontinuous;
  - **H**omogeneous;
  - **I**sotropic;
  - **L**inear . . . .
  - **E**lastic.

  - **D**iscontinuous;
  - **I**nhomogeneous;
  - **A**nisotropic;
  - **N**ot . . .
  - **E**lastic.

Under extreme loading conditions . . . . most **CHILE** materials fall into the **DIANE** classification anyway!!



A few numerical approaches are CHILE
→.. .. but most today are DIANE



Constitutive Model Overview

- Several models based on Critical State Theory including:
  - Elastic and Poro-Elastic models
  - Orthotropic Modified Cam Clay (transverse isotropic materials);

- A series of soft soil/rock models based on critical state theory. These have different levels of sophistication including:
  - Standard hardening/softening formulations;
  - Rate dependent hardening/softening formulations;
  - Combined short-term rate dependent failure and long-term creep;
  - Models which represent the evolution of the material state due to lithification;
  - Depth (porosity) dependent initial yield surface.

- Traditional strength criteria may also be employed e.g. Mohr Coulomb, Drucker-Prager, etc.

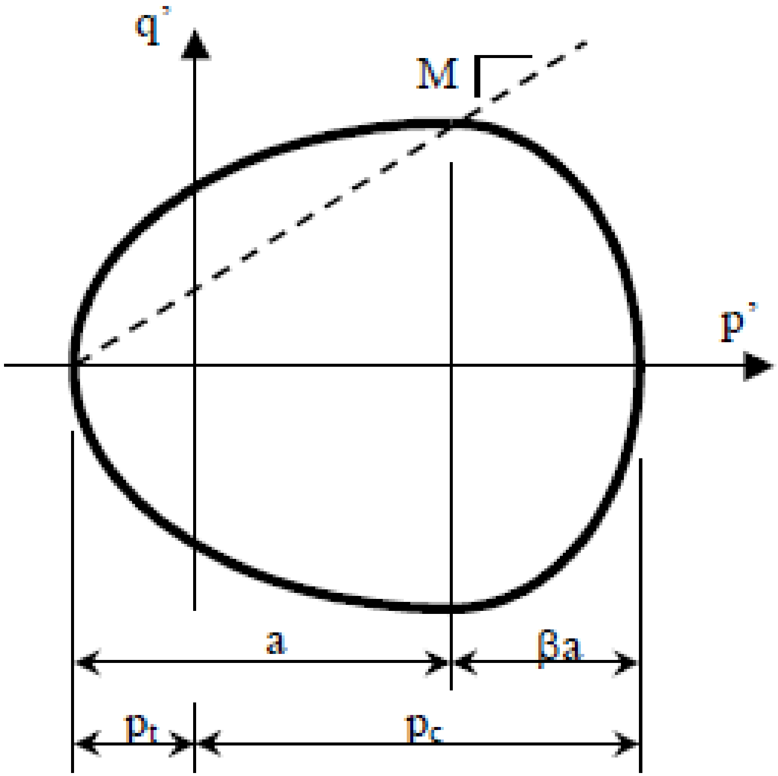
Isotropic Critical State Failure Model

- The critical state approach is often selected since its basis is derived from the experimental measurement of soils at large strain conditions;

- The isotropic Cam Clay form has been frequently employed, but with the inclusion of a number of particular aspects to allow improved representation of many different material types, including:

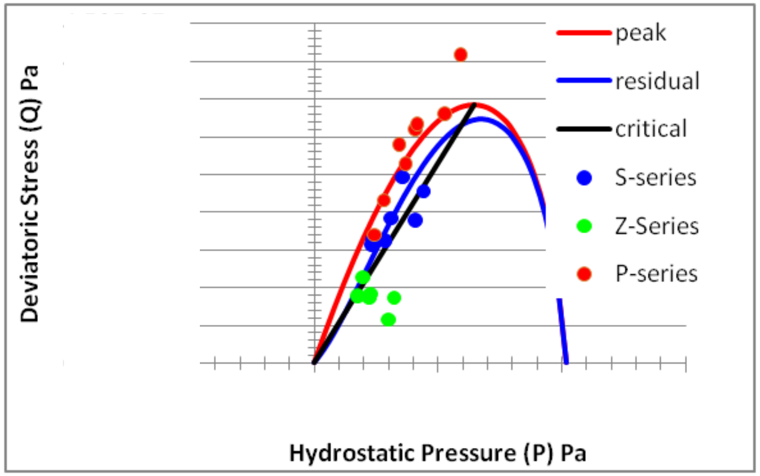
  - Fibrous materials such as peat;
  - Laminated shales;
  - Time (rate/creep) dependent materials such as chalk.

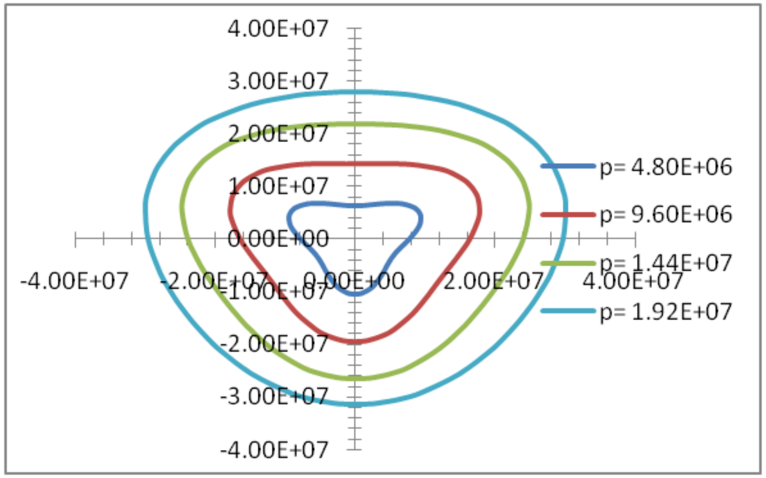
- The form of the failure surface in Pressure (P) vs Deviatoric Stress (q) space;



Isotropic Critical State Failure Model

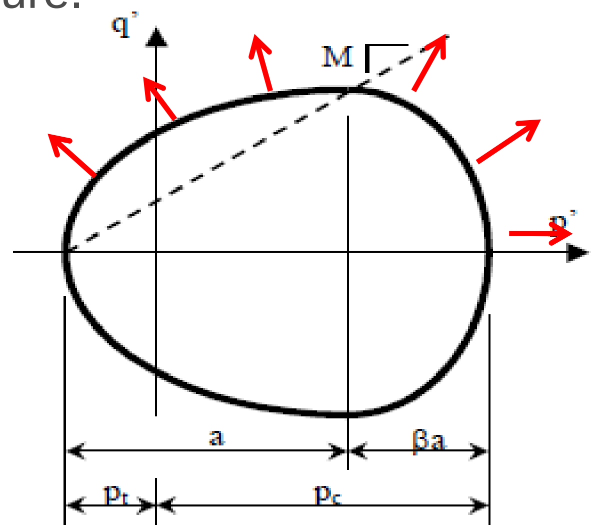
- Components of the characterised constitutive framework:
  - Transverse isotropic elasticity;
  - Orthotropic pressure dependent yielding surface based on critical state soil mechanics;
  - A smoothly varying yield surface, that is non-circular in the deviatoric plane;
  - A non-associated flow rule;
  - Mesh objectivity of the solution is achieved by incorporating fracture energy concepts;

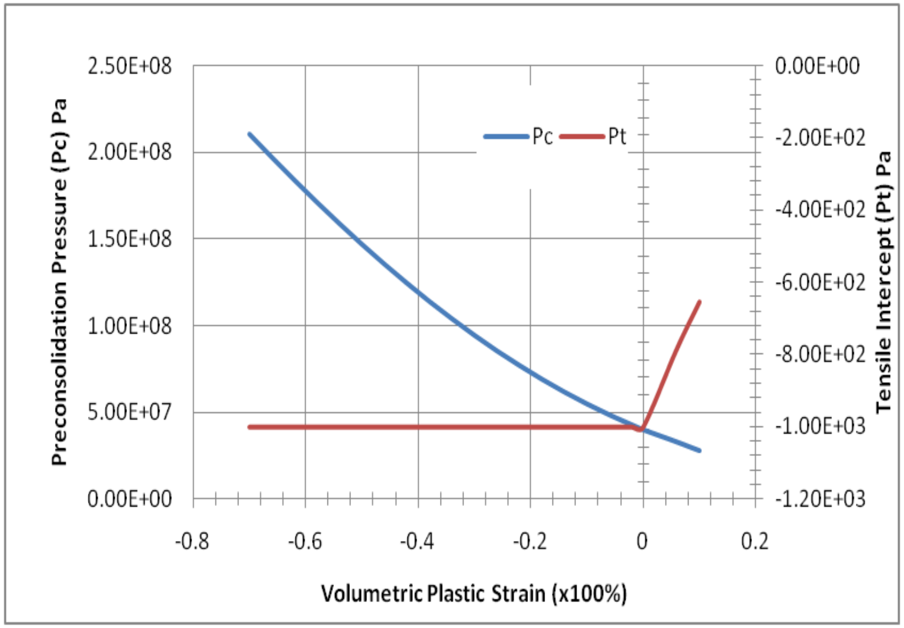




Critical State Failure Model – Strain Hardening

- Hardening of the model is achieved through expansion of the yielding surface (by variation of the two parameters Pc and Pt with respect to the volumetric plastic strain level).
- This then modifies the size of the yielding envelope based on material failure.



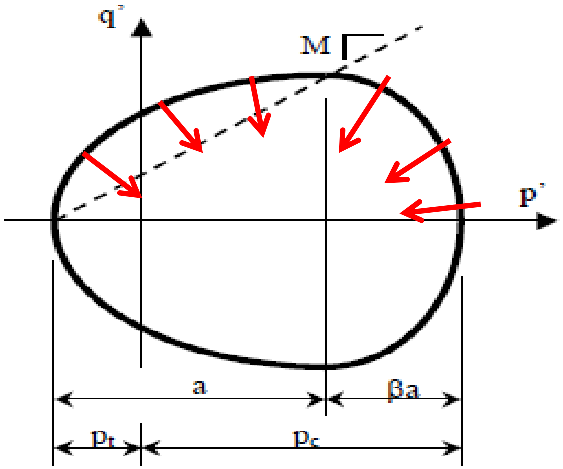


Critical State Failure Model – Creep

- The incremental creep strain is defined as

$$\Delta\varepsilon_c = \Delta t\gamma_c \;\; \Gamma^{n_c} - 1 \left(1+\frac{q}{p_c}\right)^{m_c}$$

- Defines the relationship between the rate independent pre-consolidation pressure (pc) and the pre-consolidation pressure of the rate dependent surface (pc*),



$$\Gamma = \frac{p_c^*}{p_c} = \left[1+\frac{\Delta\varepsilon_c}{\Delta t\gamma_c}\left(1+\frac{q}{p_c}\right)^{-m_c}\right]^{1/n_c}$$

- The creep strain rate is therefore nonlinear dependent on both:
  - The relative magnitude of the rate independent and rate dependent surfaces.
  - The magnitude of deviatoric stress; i.e. the creep strain rate increases at higher deviatoric stress levels.

Critical State Failure Model

- **The Isotropic Yield Surface**
- The form of the yielding function used to define the failure surface is given by:

$$\Phi\left(\sigma, \varepsilon_v^p\right) = \left(\frac{q}{M}\right)^2 + \frac{1}{b^2}(p - p_t + a)^2 - a^2 = 0$$

- Where $b = \begin{cases} 1 & p \geq (P_t - a) \\ \beta & p < (P_t - a) \end{cases}$

$$a = \frac{1}{1+\beta}(p_t - p_c); \qquad b = \begin{cases} 1 & p \geq (p_t - a) \\ \beta & p < (p_t - a) \end{cases}$$

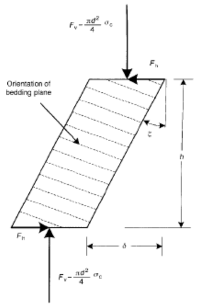
- Where P is the effective mean stress, P_c is the pre-consolidation pressure, and P_t is the tensile intercept (strength).
- The term beta is a material constant that defines the shape of the consolidation side of the failure envelope.
- The deviatoric stress q is defined in the standard manner of:

$$q = \sqrt{3J_2'} = \sqrt{\frac{3}{2}\boldsymbol{S}:\boldsymbol{S}} = \sqrt{\frac{1}{2}\sigma^T \boldsymbol{P}\sigma}$$

- Where J_2 is the second invariant of the deviatoric stress tensor **S**;
- We include the projection matrix **P** such to compute the deviatoric stress from the globally aligned stress tensor sigma.
- We define projection matrix as the following:

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{\Omega} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Gamma} \end{bmatrix} \qquad \boldsymbol{\Omega} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \ \boldsymbol{\Gamma} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Structurally Anisotropic Materials



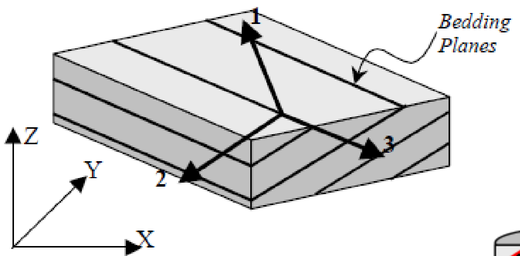
- Most typical features of such materials:
- The variation of the compressive strength with the angle between the “fabric/bedding” and loading is such that the maximum strength occurs when the loading direction is either P or N to the layering;
- The minimum stress occurs when the loading to “fabric/bedding” orientation ranges 30° - 60° where high shear induces failure on the laminations;
- The elastic properties are transverse isotropic with the Elastic Modulus normal to the “fabric/bedding” being less than the in plane;
- The elastic properties are a nonlinear function of confining pressure and effective stress.

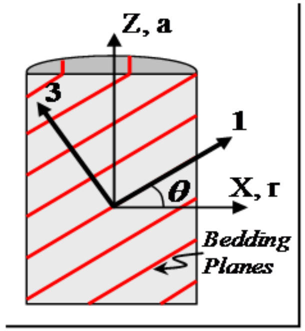
Structurally Anisotropic Materials

- The transverse isotropic nature of the material may be represented within a finite element framework in a number of ways, these include:
- Inclusion of an embedded weakness description via a smeared law, however this neglects the true compactive behaviour of the materials at higher confinement levels.
- Direct representation of the “fabric/bedding” weakness as an interface with adhesive, cohesive and frictional properties, however due to fabric dimensions this is just not practical.
- Representation of the macroscopically observed deformations using a phenomenological constitutive model based, for example orthotropic elastoplasticity.

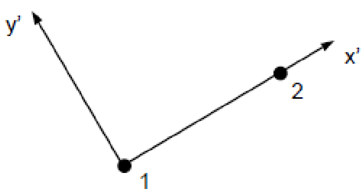
Orthotropic Transformation

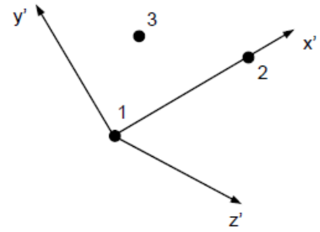
- **Orthotropic Transformation**
- We employ a local coordinate system to transform global coordinate measures (material variables) to an alternative coordinate system that coincides with material fabric directions;





- **Local Coordinate System Definition**
- We use a simple specification of three nodal points to define the system;
- Node 1 defines the origin of the coordinate system and node 2 defines the local x’ axis;
- For three-dimensional applications, node 3 defines the x’-y’ plane.





Transverse Isotropic Elasticity

- We define the elastic response of any orthotropic material through use of the locally orientated (transformed) coordinate system by using the linear Hooke's Law relationship:

$$\sigma_e^l = D_{orth}^l \varepsilon_e^l$$

- Here **D** is the matrix of the stiffness constants that interrelates the elastic stress and strain vectors respectively. The superscript *l* indicates the relationship is defined in the orientated material fabric coordinate system.
- For transverse isotropic materials like Opalinus Clay we only need specify nine elastic (stiffness) constants to provide an appropriate response;

- The required elastic constants are:
  - E_1 – the Elastic Modulus in the direction normal to the bedding plane
  - E_2 – the in plane Elastic Modulus;
  - G_{12} – the out of plane Shear Modulus;
  - V_{23} – the in plane Poisson ratio; and
  - V_{12} – the out of place Poisson ratio.

  - Of these constants it is possible to estimate the shear modulus variable G_{12} from the standard St-Venants formula:

$$\frac{1}{G_{12}} = \frac{1}{E_1} + \frac{1}{E_2} + 2\frac{\nu_{12}}{E_1}$$

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Critical State Failure Model - Orthotropy

- **The Orthotropic Extension**
- The isotropic yielding function may be extended to consider orthotropic behaviour by rewriting of the deviatoric stress equation, such that:

$$q_{orth} = \sqrt{\frac{1}{2}(\sigma')^T P_{orth}\sigma'}$$

- Leading to the orthotropic yielding function to take the modified form of:

$$\Phi(\sigma, \varepsilon_v^p) = \left(\frac{g(\theta)q_{orth}}{2M}\right)^2 + \frac{1}{b^2}(p_{orth}\sigma - p_t + a)^2 - a^2 = 0$$

- Where the projection matrix **P** is now written in orthotropic notation, such that:

$$P_{orth} = \begin{bmatrix} \Omega_{orth} & 0 \\ 0 & \Gamma_{orth} \end{bmatrix}$$

- As defined in previous notation the vector of stress variables that are aligned with the fabric of the material;
- We also make use of Hill's orthotropy notation derived for metals, in that

$$\Omega_{orth} = \begin{bmatrix} 2(\alpha_4+\alpha_6) & -2\alpha_4 & -2\alpha_6 \\ -2\alpha_4 & 2(\alpha_4+\alpha_5) & -2\alpha_5 \\ -2\alpha_6 & -2\alpha_5 & 2(\alpha_5+\alpha_6) \end{bmatrix}$$

$$\Gamma_{orth} = \begin{bmatrix} 6\alpha_7 & 0 & 0 \\ 0 & 6\alpha_8 & 0 \\ 0 & 0 & 6\alpha_9 \end{bmatrix}$$

$$p_{orth} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & 0 \end{bmatrix}$$

- Where the material constants α1- α9 control the failure value in the orthotropic framework;
- For the transverse isotropic case for the Opalinus Clay this simplifies to only α4- α9 being needed . . . .

Critical State Failure Model - Orthotropy

- The transverse isotropic condition then follows that:

$$\alpha_4 = \alpha_6, \quad \alpha_7 = \alpha_9, \quad \alpha_8 = \frac{2}{3}(2\alpha_4 + \alpha_5)$$

- This then permits the orthotropy matrices to be simplified to:

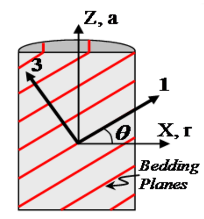
$$\Omega_{orth} = \begin{bmatrix} 4\alpha_4 & -2\alpha_4 & -2\alpha_4 \\ -2\alpha_4 & 2(\alpha_4+\alpha_5) & -2\alpha_5 \\ -2\alpha_4 & -2\alpha_5 & 2(\alpha_4+\alpha_5) \end{bmatrix}$$

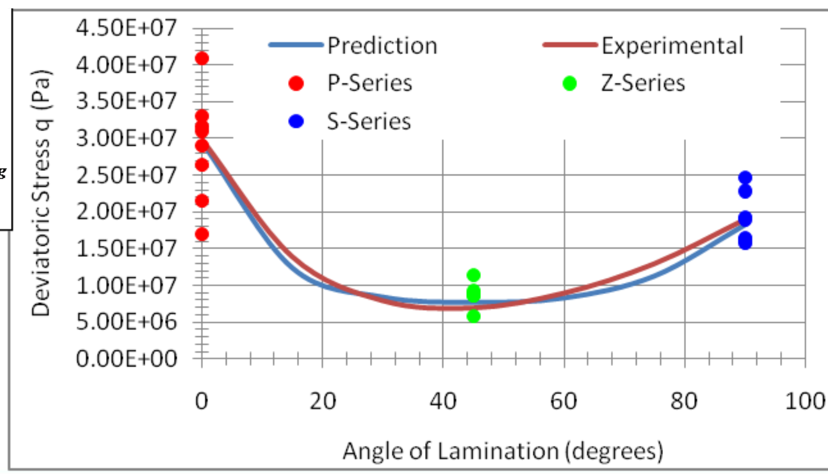
$$\Gamma_{orth} = \begin{bmatrix} 6\alpha_7 & 0 & 0 \\ 0 & 4(2\alpha_4+\alpha_5) & 0 \\ 0 & 0 & 6\alpha_7 \end{bmatrix}$$

- This then results in three unknown parameters α4-α6 that need to be derived from TXC investigations at different fabric angles.

- We define the peak deviatoric stress values in terms of the angle of lamination for a constant confinement pressure;
- We solve for the matrix of unknown parameters using an incremental (iterative) solution of the deviatoric stress equation to come up with the projection matrix $\mathbf{P}_{\mathbf{orth}}$

$$q_{orth} = \sqrt{\frac{1}{2}\left(\sigma^{l}\right)^{T} P_{orth}\sigma^{l}}$$





Porous Flow - Orthotropy

- **Orthotropy – Flow/Heat**
- We employ a transformation of the global coordinate system stress/strain variables to the material lamination aligned system;
- We employ an analogous transformation approach for the material properties required for the porous flow solution;
- Standard isotropic porous flow modelling requires specification of:
  - The isotropic permeability k_{iso};
  - The material porosity (voids ratio) & saturation;
  - The grain and fluid stiffness's;
  - The Biot constant;
- We are then able to define the orthotropic permeability in the transformed (lamination specific) coordinate system with simple scalar multiplication of the isotropic intrinsic permeability, such that

$$k_{ortho} = \begin{bmatrix} f1.k_{iso} & 0 & 0 \\ 0 & f2.k_{iso} & 0 \\ 0 & 0 & f3.k_{iso} \end{bmatrix}$$

- Where the constants f1-f3 are the factors defining the ratio to the isotropic permeability.
- The coordinate system transformation employed for the porous flow properties is identical to that for both the mechanical (strength) and thermal field types.

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